The Mind and the Limitations of Physics

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The central role of the mind is to gather and sift through information. We shall examine how this process affects our knowledge of the physical universe. Several of the limitations of classical physics, statistical mechanics, relativity theory, and quantum mechanics will be explained from this perspective.

Introduction

Over the past three centuries, science has made tremendous progress in understanding the universe around us. In the past century, science has progressed to the point where we have learned the limitations of some of the branches of science. We now know of problems that cannot be decided by a mechanical process (undecidability), processes that cannot be computed (uncomputability), and some phenomena that cannot be foreseen (unpredictability).

These limitations arise in many areas of science and mathematics (extensively discussed and classified in [YanOLR, YanoAS].) Here we shall focus on limitations of physics (e.g., [Tavel]), and see how many of the limitations in physics arise because information about the universe is organized and filtered through our finite minds.

There is a long philosophical tradition that believes our knowledge of the physical universe is dependent on the structure of our mind. This thread goes back at least to Immanuel Kant. He postulated that the human mind is not a blank slate (*tabula rasa*) affected by sensations. Rather, he believed that the human mind receives sensations through a prism of innate notions of space and time. Without these innate ideas, it would be impossible for the mind to understand the physical universe or its laws. Kant, and many others, would say that our mind makes physics possible. In this essay, we hope to show that this same mind is the cause of some of the limitations to physics.

In contrast to limitations of mathematics and theoretical computer science,[1] the limitations of physics are not inherent to the objects studied. Within physics, the limitations depend on how we learn about the objects and how we learn the laws of physics. By studying the means by which we acquire certain physical concepts, we will understand some of the limitations of physics.

The Physical Universe

Before we examine the role of the mind in understanding the physical universe, we have to understand the contents of the physical universe. What is the world made of? In order to answer this, we have to meditate on an ancient thought experiment that has puzzled philosophers for centuries: the Ship of Theseus. There was a ship that belonged to the Greek general Theseus and upon his death, the people of Athens honored him by preserving his ship at the Athenian port. With passing years, some parts of the ship rotted and were replaced. The question arises: is the ship with the replaced parts still the ship of Theseus? The obvious answer is that of course it is the same ship. Just changing parts does not make it a different ship. After all, changing the tires of a car does not change the car. (Or does it?) On the other hand, it is technically not the same ship. (Or is it?) What happens when many parts of the ship are changed over hundreds of years? Does the ship's status change? What if *all* the parts of the ship are slowly replaced? If a woodchip breaks off the ship of Theseus, is the ship still the same ship of Theseus? Is the woodchip that broke off considered part of the ship of Theseus? There are really no objective, clear-cut answers to such questions.[2]

This puzzle has nothing to do with ships. Similar questions can be asked about any physical object. One can also ask about organizations. For example, in what sense is the United States Government the same after people within the government are replaced by an election? Likewise, consider the New York Yankees baseball team that has existed for more than a century. Every few years the players change; the fans change; the owners change; even Yankee Stadium moved. Why are the Yankees still considered the same team? Similarly, how is a university the same as it changes? Every four or five years the students change; about every forty years the faculty changes; the courses and the rules are constantly changing, etc. Why should we consider these changing institutions to be the same?

Philosophers call the question of what makes an object into that object "the problem of identity." These unanswerable questions are all asking what is the essence of an object. In what sense is an object still the same after a change is made? In other words, what is an object made of? It is more interesting when we ask the same unanswerable questions about human beings. Each cell in a human body changes every seven years. Our memory and our personality are constantly changing. In what sense are you the same person as you were ten years ago? Philosophers call the conundrum of what makes a person "the problem of personal identity."

The usual lesson that one learns from the Ship of Theseus is that objects do not have persistence through time. That is, as time passes, the very essence of an object changes. However, we can go further to make more profound observations. The fact that these typical questions about the essences of objects have no objective, clear-cut answers shows us that there are no exact definitions of what an object is. There is no reason to say that a certain object contains certain parts and not other parts. Objects are not made of parts. Rather we humans accept a group of elements (while rejecting others) and we declare them to be parts of a whole object. In reality, there are no objects.[3] We combine certain sensations and state them to be caused by a single object. There are no objective parts of a collection.

Certain parts of objects seem more a part of a collection than others. If a part is physically embedded in a larger object, then it is harder to see it as an independent object. For example, an atom in the middle of a brick in the center of a pyramid at Giza has been there for millennia and will probably be there for many millennia to come. In contrast, a free part is easier to distinguish from the other parts of a collection. For example a floating particle of dust, is hard to see as connected to any whole. Nevertheless, regardless of how it seems, there are no objective parts of objects.

Once the mind accepts an object, finding that purported object's properties is not simple. Measure the length of a table to be six feet long. Is it not the same table if we take off a half a foot? What if we extend the table? The table does not have a fixed length nor should we assign it one. Similar statements can be made about the color of an object. Imagine painting the brown wood of the ship of Theseus bright pink. Is it still the same ship? Some will argue this way and some will argue the other way. The point is that the ship does not have a color. We can ask if the photons that are reflected off the ship and hit your eye are still part of the ship or are they part of the sun. Similar statements can be made about the feel of an object and the temperature of an object. In short, objects do not have properties. Rather, minds attribute properties to ensembles we call objects. The universe is bereft of properties.

A typical response to these ideas is to rebel against them. One screams: "No! Objects exist and then when we perceive them we come to classify the objects in our head as they exist in the universe. Furthermore, objects have properties associated with them and when we observe objects, we determine what properties the objects have." As obvious as these ideas sound, they are wrong. The above arguments show us that the objects that we know do not exist independently of some mind. Nor do the objects that we form have any properties. The Ship of Theseus or the desk in front of you are made of innumerable particles that, objectively, have nothing to do with each other. It is a human mind that makes them into a whole, assigns them properties, and even gives them a name. When we believe in preexisting objects and properties, we are looking at the physical universe from some metaphysically inspired point of view. There is something outside of scientific measurement which grants an element some privileged status as part of this collection rather than that collection. There is no reason to posit such an idea. There are no metaphysical names or tags on physical elements. The only things that our minds observe are sensations. Once we classify and categorize sensations, we tend to objectify them and make a whole metaphysical shish kabob out of it. While we cannot survive without that illusion, let us be aware that it is an illusion.[4]

Before we close our discussion on the Ship of Theseus, let us meditate on a thought experiment that we will need in a few pages. Look at two mountains from afar. Consider a large boulder near the top of the right mountain. Despite our philosophical knowledge that the boulder is not part of either mountain, one should obviously think of that boulder as "belonging" to the right mountain. Any sane observer would come to this conclusion. However, the scenario gets more interesting if you are informed that ten minutes before you came on the scene, a large helicopter picked up the boulder that was on the left mountain and put it on the right mountain. To emphasize, the boulder "belonged" to the left mountain for many millennia and was on the right mountain for only ten minutes. Which mountain does the boulder "belong" to now? This little thought experiment teaches us that we must incorporate time considerations when attributing properties to what we call objects.

Our Knowledge about the Physical Universe

Our Perceptions. How does a mind make sense of all the sensations that it is bombarded with? The mind taxonomizes. In fact, that is its main role. The mind has to group together various sensations, classify the sensations and categorize them. Part of gathering information is excluding some, that is, we act like a sieve and only retain some of the information. There are no objects, and even if you accept a collection as an object, there are no properties of the object. It is our mind that puts together objects and also accepts that these objects have certain measurements. When I say the table before me is yellow, I am really saying that those parts that I declare are part of what I call a table also give me the sensation of yellow.

Cognitive scientists have been working to understand these processes that the mind performs to make sense of sensations. They study the "binding problem," how one binds a property to an object. They are quite adept at making physiological models of how these processes are done. They also find where in the brain these tasks are performed and how people with lesions or anomalies in those areas perceive the world. It is not our task to criticize or contribute to the important work of cognitive scientists. There is, however, a certain philosophical difference between our point of view and theirs. Most cognitive scientists would say that objects exist with properties and they are interested in how the mind finds these preexisting objects and their properties. For us, these objects and their properties do not exist in the first place. The mind brings together sensations as if objects exist with their concomitant properties.

We are not purporting something weird or post-modern here. No one is claiming that everyone's perceived reality is different and subjective. The brains of different people are physiologically very similar and their educations are compatible so that they mostly agree on what is out there.

Our Laws of Physics. This ability of the human mind to gather, exclude, and categorize does not only work with our sensations. This facility is also used to gather, exclude, and categorize results of experiments so that we can formulate the laws of physics. Physicists look at many results of experiments and then try to use these results to distill the laws governing the phenomena being studied. The main criterion for such gathering is symmetry. Two different experiments are deemed symmetric when, even though they differ in some aspect, the results of the experiment are the same, e.g., two experiments performed in different places are treated as one phenomenon. They are said to have *spatial symmetry*. We can also talk of the same experiment performed at different times and in different orientations. These symmetries are called *temporal symmetry* and *rotational symmetry*, respectively. More complicated types of symmetry, such as *gauge symmetries*, explain many other laws of nature. The Standard Model, which describes all subatomic particles and their interactions, can be explained with such symmetry considerations [Yan2].

It is not that these laws of physics actually exist independent of the human mind. It is the human mind that gathers and classifies the phenomena. The laws derived from this procedure have the feel of being objective only because the phenomena are gathered when they are true in any place, in any time, in any orientation, etc. They feel objective because that is the way we gathered them. The objectivity is an illusion.

It should be noted that the laws that we conjure up with this procedure do not constitute the ultimate truth because we do not look at all possible phenomena. Rather, we only look at the subset of observed phenomena. In detail, there are two sets of phenomena. There are (i) all observed phenomena, which are included in (ii) all possible phenomena. The physicist classifies the phenomena in (i) and comes up with grand statements about all the phenomena in (ii). When phenomena which were previously in (ii) and not in (i) make their way into (i), i.e., when previously unobserved phenomena become observed, and there are differences in results, these phenomena are anomalies. When enough anomalies accrue, a revolution is in order [Kuhn]. It is precisely the possibility of such revolutions that makes all purported laws of physics tentative. Rather than being true, such laws are awaiting falsification [Popper]. This is a limitation of our ability to know the ultimate laws governing our physical universe.[5]

Classical Physics

Mechanics. This field of physics deals with everyday objects. That is, it deals with several (not many --- which is the domain of statistical mechanics) large (not microscopic ---which is the domain of quantum mechanics) objects moving at normal speeds (not close to the speed of light --- which is the domain of relativity theory). These are your basic everyday phenomena.

The problem with classical mechanics is that its laws are supposed to work on large-scale objects but there does not exist any large-scale objects. All large-scale objects are created by us and we idealize them to make them applicable to the laws. In fact, the usual laws of classical mechanics are not applicable [Cart1, Cart2, Giere]. Consider Isaac Newton's famous law that describes the attractive force between two objects. In order for Newton's law to apply, the objects need to be exactly spherical symmetric, homogeneous (mass must be evenly distributed), devoid of any electrical charge or magnetic attraction, not traveling near the speed of light (lest the laws of special relativity take over), nor can they be too small (lest quantum effects come into play), etc. The law also (wrongly) assumes that there are no other objects in the universe. Any other object would influence the forces in incomputable ways. And the list goes on. In summation, it is safe to say that there never were two bodies that exactly satisfied the requirements for Newton's laws to be applicable. Rather, the physicists must make controlled experiments and ignore all these other effects. By selecting the phenomena, she idealizes the actual world to find applications of the laws. Without this false idealization, the laws would have no applications.[6]

Chaos. Newton taught us how to deal with two idealized large-scale objects. What about three objects? The dynamics of such a system has too many changing forces to deal with. This is called *the three-body problem.* Our mind and our mathematics is generally incapable of making any long-term predications on such systems. Let us summarize the limitations of the predictability of our universe: Physics can easily deal with an empty universe or a universe that contains one particle. Newton taught us how to deal with two idealized objects. We are clueless at dealing with three or more idealized objects. However, physicists tell us that there are about 10^{80} particles in the physical universe. Our universe is extremely unpredictable.

Thermodynamics and Statistical Mechanics

Probabilistic Laws. The human mind cannot deal with many objects. When there are many objects in one system, there are no laws that can exactly describe the working of all the objects. Instead, physicists tell us we can make probabilistic statements about ensembles rather than deterministic statements about individuals. We are ignorant of the actions of individuals.

The problem with probabilistic laws is that they are only a product of our mind and our gathering faculty. Consider the probabilistic law that says that when you flip a coin, there is a fifty-fifty chance of getting a heads. When you actually flip a coin, it will come up either heads or tails. There is nothing fifty-fifty there. Each flip is an individual event and there is nothing probabilistic one can say about it. Flip a coin 10 times. It is actually improbable ($p=0.24$) that someone would get exactly five heads and five tails. One has to flip a coin many times to get some type of fifty-fifty ratio[7]. In fact, to get the exact probabilistic statement one has to idealize and gather an (unattainable) infinite number of coin flips. The probabilistic rule is not really applicable in the real world.[8]

Irreversibility and Entropy. When dealing with a large amount of components, there is a possibility of a process being irreversible. In contrast, all the processes in classical physics are reversible. In detail, there is a notion of entropy, which is a measure of disorder. The second law of thermodynamics states that the entropy of an isolated system will probably increase. That is, disorder will probably increase.

A deep question is how can particles that all follow reversible laws of physics give rise to processes that appear irreversible. The answer is that these processes are irreversible to us and the way our mind learns about the universe. If we were minute creatures looking at several components interacting, we would not see irreversibility. Similarly, if we had a significantly larger mind, we might find it easier to follow all the parts of the system. Irreversibility is an illusion that depends on the mind looking at it. One of the leading physicists of the last century, Max Born, stated this very clearly: "Irreversibility is a consequence of the explicit introduction of ignorance into the fundamental laws."[9]

Relativity

Special Relativity. One of the consequences of electrodynamics is that the speed of light (in a vacuum) is a fixed finite amount. Let us call this amount c. In contrast to classical physics where when one measures the speed of an object, one has to take into account their own movement, the speed of light is always measured at the speed c regardless of the velocity of the observers. Special relativity deals with the consequences of this fact.

Our minds do not have built in measuring devices. All our measurements are comparisons. We use a watch to measure the duration of an event, we use a ruler to measure the length of an object, and we use a watch and a ruler to measure the velocity of a moving object. The problems arise when we use a measuring device in one frame of reference to measure an event or object in another frame of reference that is in relative velocity with the first. The values for measurements done in different frames of references can be different. *Time dilation* is when durations are measured to be different. *Space contraction* is when objects are measured to be different sizes. The main point is that events do not have objective durations, objects do not have objective lengths, and objects do not have objective velocities. The properties depend on how they are measured, and hence they are not objective or inherent properties.

General Relativity. Special relativity deals with observers moving in unaccelerating frames of references. Einstein extended these ideas to general relativity where frames of reference can be accelerating. He extended the symmetry considerations of special relativity to these frames of reference. One of the central ideas is the *Equivalence Principle*, which states that there is an equivalence between the feeling of gravity and the feeling of acceleration. A consequence of this is that the notions of length, duration, and speed depends on where one is in the universe. If you are far away from any mass, you will get some measurements. In contrast, if you are near a black hole, you will get other measurements. This variability of such fundamental notions can be seen as a limitation of physics.[10]

Quantum Mechanics

Superposition. The most shocking and counterintuitive notion in all of science is the idea of superposition. The simplest example of this is an unobserved subatomic particles not having a single "position," but having many positions or a "superposition." In the well-known double-slit experiment, a particle does not go through the right slit or the left slit. It goes through both slits simultaneously. Position is not the only property that possesses this strange characteristic. Many energy values and spin values also have superposition. In short, unobserved quantum systems do not have definite properties. Another way to say this is that they do not have *value definiteness*. As strange as this sounds, it is a scientific fact that has withstood experimental confirmation. There is also an unassailable mathematical theorem proving the lack of value definiteness called the Kochen-Specker Theorem.

While there is no way we can make the mystery of superposition go away, from the perspective supplied above, it is slightly less strange. We saw that objects do not have properties when they are unobserved. It is only when we observe objects that we assign them properties. This is even more so when we have isolated microscopic objects. The quantum worldview and our object-less worldview are in accord.

Measurement Problem. Unless one had too much Tequila, one usually does not see single objects in more than one position. The very reason why superposition is so strange to us is that it is not observed. Physicists inform us that when we make an observation or a measurement of a quantum system, the superposition "collapses" to a position with definite values. Why such a collapse happens is called *the measurement problem*.

Traditionally, several possible solutions are given for this problem. Each of the purported solutions has its own difficulties. (i) Consciousness. John von Neumann and Eugene Wigner suggested that the very act of measurement by a conscious being causes the collapse. After all, no conscious being ever reported observing a superposition. It must be that consciousness causes the collapse. The problem with this is that it is not clear how consciousness is responsible for this task. This solution makes consciousness into some metaphysically powerful device. (ii) Multiverse. A stranger solution is that when a measurement is made, the superposition does not collapse, rather the universe splits into many different versions where each version has one of the positions. In every universe there is an observer who sees the position to which it collapsed. The problem with the multiverse is that there is no solid evidence of such a multitude of universes. (iii) Decoherence. A more contemporary solution is that somehow when the quantum system interacts with any macroscopic system, the superposition "decoheres" into a position because all macroscopic systems have position and not superposition. The problem with this is that, as we explained above, macroscopic systems are all in our mind. There is no objective size which separates the macroscopic from the microscopic.

From our perspective, the measurement problem has a simpler solution. We saw that objects do not have properties until a mind observes them and classifies them. This is what happens when a person makes an observation of a quantum system. The system goes from being hazy, nebulous and without properties to a system with properties. Notice that our solution has some commonalities with the consciousness solution. We agree that a mind has to observe an object. Although we are not sure what the purpose of consciousness is in this process. Also notice that our solution has some commonalities with the decoherence solution. As we saw, an object that is integrated with a large ensemble is apt to share the properties of the larger ensemble. Hence, when an object integrates with a large ensemble, it is easy to tell what its properties are.

Researchers in child development tell us that infants have to learn that objects exist even when they are not observed, that is, they learn *object permanence.* Quantum mechanics tells us that when we are not observing objects, they do not really exist. Modern physics demands that we unlearn the object permanence that we learned as infants.

There are some obvious questions about the collapse of a superposition. When measuring a system, to which position does the superposition collapse? The answer is that it depends on which property you are measuring. If you measure one property and then another you might get a different answer than measuring the properties in the other order. This is the content of Heisenberg's renowned Uncertainty Principle. There is also a question as to what are the probabilities of a superposition collapsing to a particular position. Here we come to conclusions similar to those we saw with the probabilistic notions in statistical mechanics. However, in quantum mechanics, there does not seem to be any deterministic reason why one position happened rather than another. It is random and beyond our ability to predict.

Entanglement. Researchers have shown that different parts of the universe are intimately connected in ways that were once unimaginable.[11] The simplest example of such phenomena is when a particle with no spin breaks apart into two particles that have spin. As long as the two particles are not observed they are each spinning in a superposition of both ways. The universe has a conservation of spin law, which says that the sum of the spins of the two daughter particles must be equal to the spin of the starting particle, i.e., zero. A consequence of this conservation law is that even if the particles are millions of light years apart, when one is measured and collapses to one type of spin, the other automatically must collapse to the opposite type of spin. That is, the measurement of one particle affects the other particle even though it is millions of miles away.

This too is less strange from our perspective. When we measure the spin of a particle, we have to take into account the history of the particle. Did the particle have a partner particle in its history? If so, they will be entangled. While it is strange to look at two entangled particles at an instance of time, it is more understandable if we look at the two particles through an *interval* of time. We have to take into account all of the space and time when the particles existed in order to understand the consequences of a measurement. This is similar to the thought experiment of the boulder on top of the mountain. We have to look at the boulder through history and we have to look at the particles through history. When we consider the boulder on the mountain through history, we get a better view of the boulder's properties. Similarly, when we consider the pair of particles through history, they seem to be more part of a single system rather than separated systems. The Ship of Theseus puzzle and quantum mechanics are both telling us that the typical way we look at objects within infinitesimal slices of time is wrong. In a sense, entanglement confirms the conclusion we learned from the Ship of Theseus: there is more to connectedness than proximity.

Entanglement brings to light another limitation of our knowledge of the physical universe. Since any part of the universe can be connected to any other part of the universe, it is inherently impossible to understand one part of the universe by itself. That part might be affected by other parts of the universe. This is a severe restriction of the doctrine of reductionism, which is a central dogma of the scientific process.[12]

We have come to the end of our journey. Our goal was to point out how some of the limitations of our knowledge of the physical universe can be seen as limitations of the minds that study the universe. We have to be cognizant that physics is a human endeavor created by finite, flawed beings attempting to know ultimate truths. The data sets that we examine are partial, the theories that we come up with are tentative, and the equations that we find are incomplete. We are not promoting some postmodernist notion that science is unreal or fictitious. Rather, there exists limitations to the ways the human mind finds and describes the world around us. In addition to studying the universe, we must also study *the way* we study the universe.[13]

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Technical Endnotes

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[1] In mathematics, the objects of study are not physical objects that we come to know through physical sensations. Mathematical limitations are inherent in the definitions of the structures. For example, Évariste Galois (and others) have shown that there are some polynomial equations of degree five or higher which cannot be solved by standard methods. This limitation is a consequence of the definitions of "polynomial equation" and "standard methods."
Another type of limitation that is found in mathematics follows from self-reference. The paradigmatic example of these is Kurt Gödel's famous Incompleteness Theorem, which shows that basic number theory can be self-referential and there exists mathematical sentences which state "I am unprovable." Such statements are unprovable (otherwise the system would be inconsistent) and hence true. These limitations are inherent to the subject at hand and do not seem to have anything to do with the mind of the person understanding it. We will point out self-referential limitations when they arise.
The limitations of theoretical computer science fall into two categories. (i) Computability theory --- which determines what tasks a computer cannot perform --- has self-referential limitations. The paradigmatic example is the Halting Problem, which asks if there exists an algorithm that can determine if any given algorithm (with input) will halt or go into an infinite loop. Since there are algorithms dealing with algorithms, there is self-reference. Alan Turing showed that no such algorithm can exist. (ii) Complexity theory --- which determines what tasks a computer cannot perform efficiently ---- has limitations that arise from the size of the problems. The paradigmatic example of such a problem is the Traveling Salesman Problem. It would take trillions of centuries to solve this problem for large inputs.

[2] One of the philosophers that wrote about the Ship of Theseus was Thomas Hobbes. He asked what would happen if, rather than disposing of the rotted pieces of the ship, the discarded pieces were stored in a warehouse. Then, after many years, when every single part of the ship has been replaced, someone sneaks into the warehouse and puts together the old rotted parts of the ship. Which ship is the "real" Ship of Theseus? Is it the totally refurbished ship in the dock or the ship with the original pieces in the warehouse? The point we are making is that it is not very clear what makes up the Ship of Theseus.

[3] Others have written on the same idea. Peter Unger [Unger] uses the sorites / heap paradox to bring this idea forward. Ted Sider [Sider] explains it in the simplest way. He writes that when he contemplates {a, b, c} he agrees that three things exist: a, b and c. He does not believe that four things exist: a, b, c, and {a, b, c}.

[4] One might criticize this view of the world being without objective objects or properties as a bleak, featureless universe. Such criticism does not perturb us. In fact we take the view afforded here in a positive humanistic way. We cognitive beings have an important role in creating the world around us. The illusion --- of there being objective objects with properties --- is itself interesting.

[5] The laws of mathematics are also formulated using symmetry considerations. Mark Zelcer and I [YZ] describe the types of symmetries one needs to consider in order to define the truths of mathematics. It is because mathematics is defined using these symmetries that mathematics has

the feel of being eternal and objective. In [Yan1] I show that this conception of mathematics can help explain Eugene Wigner's mystery of the "unreasonable effectiveness of mathematics in the natural sciences" [Wigner]. We explicate how the ethereal mathematics works so nicely in describing the workings of the physical universe. It is shown that the symmetries of physics are subsets of the symmetries of mathematics. In other words, it makes sense for the mathematics that we define to be so effective for the physics we find.

[6] Another example of this idealization of physics was when Galileo Galilei studied gravity by observing balls rolling down ramps. The laws that he found assume that the balls are perfectly, spherically symmetric, the ramps are perfectly flat and frictionless (an impossibility), there is no air resistance, etc. All these are physical impossibilities.

[7] Another example where these ideas are clearer is a Galton board. This is a set-up where there is an ensemble of balls that fall through a grid of obstructions. The path of each ball is essentially random, and yet when we look at all the balls together, they form a bell curve. Notice if we only looked at a few of the balls, there would be a more random, unstructured pattern. It is only when we artificially unite them as one that we see the bell curve.

[8] The very notion of temperature, which is at the core of thermodynamics, is defined as the average energy of an ensemble of elements. Such an ensemble exists only in our minds. It is not really out there.

[9] Since increasing entropy is the only property that changes with time, people define the arrow of time as the direction which entropy increases. Some researchers take this idea further. If increasing entropy is an illusion, then maybe the entire notion of time passing is also an illusion. (To me, this is going too far.)

[10] There is also an element of self-reference in relativity theory. Notice that the size of an object being measured depends on the relative speed of the frame of reference that contains the experimenter. This means that the results of an experiment depends on the object and the experimenter. Another way to say this is that the experimenter is part of the experiment.

[11] These phenomena were first noticed by Albert Einstein and Erwin Schrödinger in the 1920's. They were clarified with the famous EPR thought experiment of 1935, and John Bell's celebrated inequality in 1964. The phenomena was experimentally confirmed by Alain Aspect in 1982.

[12] There is an element of self-reference in quantum mechanics also. The easiest example where this is seen is with photons. It is well known that a photon acts like a wave (if one does a double-slit experiment) and a particle (if one does a photoelectric experiment). This means that a photon's resulting action will depend on which experiment is done. The experimenter and her choices are part of the experiment.

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